FINITE ELEMENT ANALYSIS APPLIED TO GYROELECTRICALLY LOADED WAVEGUIDING STRUCTURES

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ABSTRACT

A finite element formulation has been used to obtain the dispersion relation for a single dielectric-semiconductor interface bounded by two perfectly conducting planes. This system represents a suitable canonical problem for the design of non-reciprocal devices such as circulators, isolators, and phase shifters. The finite element solution for the dispersive behavior was compared against the exact solution for the lowest real branches, and excellent agreement was found between the two.

INTRODUCTION

In recent years interest in various waveguiding structures in the millimeter (mm) and submillimeter (smm) wavelength, i.e., 100-1000 GHz, has been growing. The development of this technology requires a parallel development of more accurate computational techniques. The finite element method provides an attractive approach to the problem of obtaining the dispersive behavior of the waveguide. The effectiveness of this method has led many researchers to apply it to different electromagnetic field problems, (1).

The single dielectric-semiconductor interface model considered in this paper is a suitable canonical problem for the design of non-reciprocal devices such as circulators, isolators, and phase shifters. Propagation characteristics of structures used to obtain such circuit functions have been analyzed in (2-3).

THE WAVEGUIDING STRUCTURE

Consider the dielectric-semiconductor single interface sided by two perfectly conducting planes shown in Fig. 1, with a superimposed finite element mesh. A high quality n-type GaAs material has been taken as the substrate for the semiconductor region. The system is assumed to be exposed to a uniform d.c. magnetic field along the y-direction, with time-harmonic wave propagation in the z-direction. Only TM modes will be considered in the present analysis since TE modes do not have significant interaction with the semiconducting material.

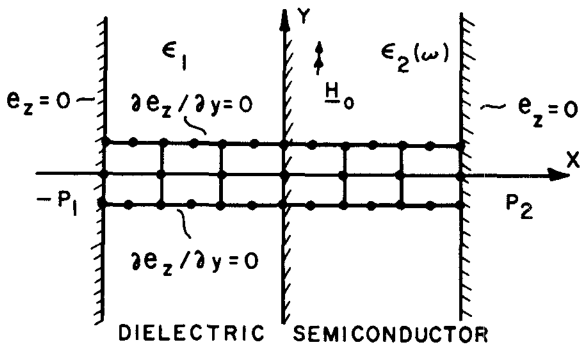

Fig. 1 - Dielectric-Semiconductor Single Interface

We take the permeability μ_0 to be a constant for both regions. The permittivity ε is a scalar constant for the dielectric medium, but becomes a tensor for the semiconducting material. For a biasing magnetic field in the y-direction, the dielectric tensor takes the following form, (3).

$$\varepsilon_2(\omega) = \begin{bmatrix} \xi & 0 & -j\eta \\ 0 & \zeta & 0 \\ j\eta & 0 & \xi \end{bmatrix} \qquad (1)$$

where

$$\xi = \varepsilon^{(0)} - \frac{\omega_p^2(\omega-j\nu)}{\omega[(\omega-j\nu)^2-\omega_c^2]}$$

$$\eta = \frac{-\omega_p^2\,\omega_c}{\omega[(\omega-j\nu)^2-\omega_c^2]} \qquad \zeta=\varepsilon^{(0)} - \frac{\omega_p^2}{\omega(\omega-j\nu)}$$

and ω_p is the plasma frequency, ν the collision frequency, and ω_c the cyclotron frequency, $\omega_c = eB_0/m^*_c$. For the isotropic case, $\omega_c = 0$ and the tensor elements reduce to

$$\xi=\zeta=\varepsilon^{(0)} - \frac{\omega_p^2}{\omega(\omega-j\nu)} \text{ and } \eta = 0$$

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From Maxwell's equations, uncoupled two-dimensional partial differential equations were derived for e_z. These equations have the common form

$$\frac{\partial^2 e_z}{\partial x^2} + M_1 \frac{\partial^2 e_z}{\partial y^2} + M_2 e_z = 0 \qquad (2)$$

where $M_1 = \frac{\zeta(\gamma^2+\omega^2\mu_0\varepsilon_0\xi)}{\xi(\gamma^2+\omega^2\mu_0\varepsilon_0\zeta)}$,

where $M_2 = \gamma^2 + \frac{\omega^2\mu_0\varepsilon_0(\xi^2-\eta^2)}{\xi}$

for the semiconducting medium and $M_1 = 1$, $M_2 = \gamma_1^2+\omega^2\mu_0\varepsilon_0\varepsilon_1$ for the dielectric medium. Moreover the magnetic field component h_y is given in terms of e_z by

$$h_y = \left(\frac{-j\omega\varepsilon_0\xi}{\gamma^2+\omega^2\mu_0\varepsilon_0\xi}\right)\left(\frac{\partial e_z}{\partial x} + \frac{j\eta\gamma}{\xi} e_z\right) \qquad (3)$$

in the semiconducting medium and

$$h_y = \left(\frac{-j\omega\varepsilon_0\varepsilon_1}{\gamma^2+\omega^2\mu_0\varepsilon_0\varepsilon_1}\right)\frac{\partial e_z}{\partial x} \qquad (4)$$

in the dielectric medium.

The dielectric-semiconductor interface was taken to be lossless (ν=0). Assuming there are no y-variations ($\partial/\partial y$=0), a TM mode solution exists in the vicinty of the interface (components h_y, e_x, e_z). The electromagnetic field boundary condition at the perfectly conducting plane requires that nxE=0. The dispersion relation is obtained from imposing this boundary condition and requiring continuity of e_z and h_y at the interface. The result is

$$\left(\frac{\varepsilon_1}{K_1}\right)\text{Coth}(K_1P_1) = \left(\frac{\xi K_2}{\gamma^2+\omega^2\mu_0\varepsilon_0\xi}\right)\text{Coth}(K_2P_2) - \left(\frac{j\eta\gamma}{\gamma^2+\omega^2\mu_0\varepsilon_0\xi}\right) \qquad (5)$$

with

$$K_1^2 = -\gamma^2 - K_0^2\,\varepsilon_1;\; K_2^2 = -\gamma^2 - K_0^2\,\varepsilon_e(\omega)$$

Here $\gamma = \alpha + j\beta$ is the complex propagation constant, and $K_0^2 = \omega^2\mu_0\varepsilon_0$, $\varepsilon_e(\omega) = \frac{\xi^2-\eta^2}{\xi}$. Furthermore, α is the attenuation constant, β is the phase constant, ε_0 the permittivity of vacuum, μ_0 the permeability of vacuum ε_1 the relative dielectric constant of the dielectric medium, $\varepsilon^{(0)}$ the static dielectric constant of the semiconducting medium, ε_e the effective dielectric constant of the semiconducting medium, P_1 the width of the dielectric medium, and P_2 the width of the semiconducting medium.

FINITE ELEMENT FORMULATION

The finite element mesh for this problem is shown in Figure 1. The finite element interpolating functions utilized were those for the eight-noded isoparametric quadrilateral element, (4). Both the dielectric and semiconducting regions were divided into three equally-spaced elements. The finite element equations were generated from equation (2) by means of the Galerkin formulation

$$\iint_A \left(\frac{\partial^2 e_z}{\partial x^2} + M_1 \frac{\partial^2 e_z}{\partial y^2} + M_2 e_z\right)N_i(x,y)\,dydx=0 \qquad (6)$$

where the $N_i(x,y)$ are the finite element interpolating functions, and the index i ranges over those nodal points at which the value of e_z is not specified by the boundary conditions. We now introduce the finite element approximation $e_z = [N(x,y)]\{e_z\}$, where $[N(x,y)]$ is the row vector of interpolating functions and $\{e_z\}$ the column vector of nodal point values. An application of the divergence theorem now yields the finite element equations in the form

$$[A]\{e_z\} - \oint_{\bar{B}} \left(\frac{\partial e_z}{\partial x} n_x + M_1 \frac{\partial e_z}{\partial y} n_y\right)N_i\,ds=0 \qquad (7)$$

The i,j-th element in the coefficient matrix [A] is given by

$$A_{ij} = \iint_A \left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + M_1 \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} - M_2 N_i N_j\right)dxdy \qquad (8)$$

and is evaluated by 3x3 point Gaussian integration. The line integral in equation (7) is to be evaluated around the boundary $\bar{B}$ of the finite element mesh, and (n_x, n_y) are the components of the unit normal vector to $\bar{B}$. When assembling the finite element equations from element contributions, as is usually done, the integral is to be evaluated around the boundary $\bar{B}_e$ of each element.

The boundary conditions $e_z = 0$ at $x = -P_1, P_2$, as well as the condition $\partial e_z/\partial y=0$ on the top and bottom sides of $\bar{B}$, can easily be shown to lead to the vanishing of the line integral along these portions of $\bar{B}$. Moreover interelement compatability conditions between adjacent elements in the same medium lead to a vanishing net contribution when the line integral is evaluated over the common interelement boundary. This is not true, however, along the interface between the dielectric and semiconducting regions, where special care must be taken.

Let n be one of the three values of the index i corresponding to the three interface nodes. Further denote the values of e_z in the dielectric and semiconducting regions of $e_z^{(1)}$ and $e_z^{(2)}$ respectively. Along the interface we must have both continuity of e_z, $e_z^{(1)} = e_z^{(2)}$, and the continuity of h_y. The latter condition, from equations (3) and (4), gives that

$$\frac{\partial e_z^{(1)}}{\partial x}\Big|_{x=0} = \left(R_1 \frac{\partial e_z^{(2)}}{\partial x} + R_2 e_z^{(2)}\right)\Big|_{x=0} \equiv C \qquad (9)$$

where $R_1 = \frac{\xi(\gamma^2+\omega^2\mu_0\varepsilon_0\varepsilon_1)}{\varepsilon_1(\gamma^2+\omega^2\mu_0\varepsilon_0\xi)}$ and $R_2 = (\frac{jn\gamma}{\xi}) R_1$. We now write the n-th finite element equation separately for both the dielectric and semiconducting regions. These are, respectively (with summation convention implied)

$$A^{(1)}_{nj} e^{(1)}_{zj} = Cd_n$$

$$A^{(2)}_{nj} e^{(2)}_{zj} - \left(\frac{R_2}{R_1}\right) b_{nj}\, e_{zj}^{(2)} = \frac{Cd_n}{R_1} \qquad (10)$$

where $d_n = \int_{-\alpha}^{\alpha} N_n\,(o,y)dy$ and $b_{nj} = \int_{-\alpha}^{\alpha} N_n\,(o,y)N_j\,(o,y)dy$. Eliminating C between equations (10) and requiring continuity of e_z at the interface now yields for the n-th equation

$$(A^{(1)}_{nj} + R_1 A^{(2)}_{nj} - R_2 b_{nj})\, e_{zj} = 0 \qquad (11)$$

Equations (11) represent the finite element equations corresponding to the interface nodes. The remaining finite element equations have the form of equation (7), with the line integral vanishing for these equations. When assembled, the finite element equations have the form $[A^*]\{e_z\} = 0$. A nontrivial solution then requires that

$$|A^*| = 0 \qquad (12)$$

which represents the finite element dispersion equation for the problem. Given a value of ω, one may obtain corresponding values of γ by standard numerical root-finding techniques.

RESULTS AND CONCLUSIONS

Equations (5) and (12) represent respectively the exact dispersion relation and the finite element approximation. Both equations were solved numerically using a standard technique, the bisection method, for the following physical constants: $\varepsilon^{(0)}=13$, $\varepsilon_1=1$, $\omega_p=10^{13}$ rad/sec, $\omega_c=10^{12}$ rad/sec, $P_1 = 80$ µm, $P_2 = 100$ µm. Figures 2 shows the lowest positive and negative real branches of the dispersion spectrum obtained from the exact and finite element dispersion equations for $|\beta| < 2$, for a normalized propagation constant defined by $\bar{\beta} = P_2\beta$ and a normalized frequency given by $\bar{\omega} = \omega P_2/c$. It can be seen that the agreement between the two is excellent. In fact, three-digit agreement was typically noted in the numerical results.

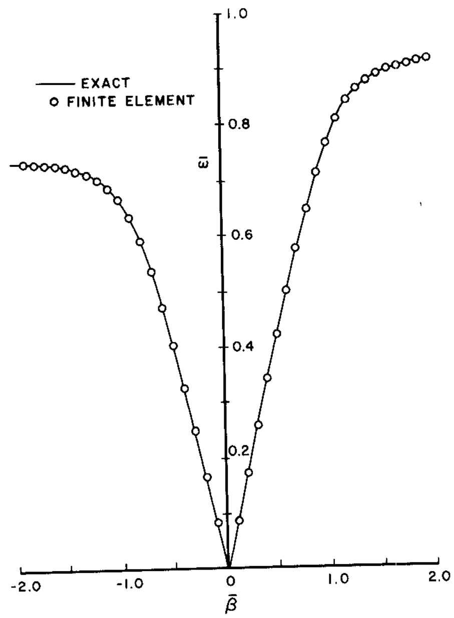

Fig. 2 - Exact and Finite Element Dispersion Spectra

Figures 3 and 4 show the distribution of e_z at two points each along the positive and negative branches, one taken in the linear portion of each curve and the other in the flattened portion of the curve. The value of e_z at the interface was normalized to unity. Again excellent agreement between the results of the exact solution and the finite element approximation may be noted.

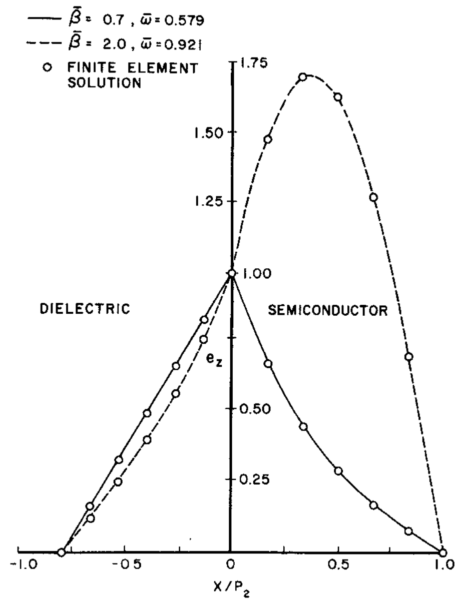

Fig. 3 - Distribution of e_z at Two Points Along Positive Branch

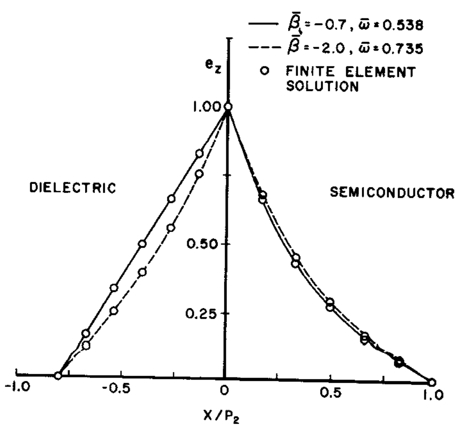

Fig. 4 - Distribution of e_z at Two Points Along Negative Branch

To summarize, we have applied the finite element method to the problem of obtaining the dispersion characteristics of a relatively simple, one-dimensional waveguiding structure. Excellent results were obtained. The primary advantage of the finite element method is, of course, its ability to treat problems of practical interest involving complicated two - dimensional geometries and correspondingly complicated electric and magnetic field distributions. The results given here indicate that the finite element method holds great promise for these applications, and in particular for the analysis of complex gyroelectrically and gyromagnetically loaded waveguiding structures.

ACKNOWLEDGEMENT

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